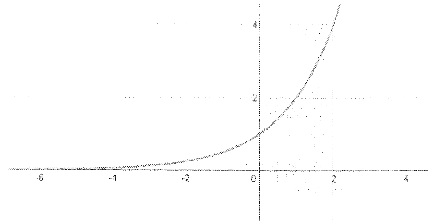
Core Math Ch. 4 Key Concepts

Algebra 2 / Trig

Ch4 Exponential and Logarithmic Functions

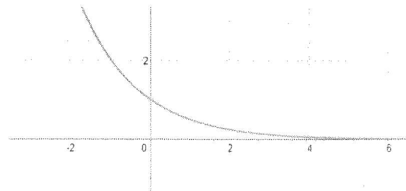
- **Exponential Functions**- When the variable is the exponent. $f(x)=b^x$ where b is positive and not 1.
- **Exponential Graphs:**
  - **When b>1 we have exponential growth.**

    End Behavior: As $x \to \infty$, $f(x) \to \infty$. As. $x \to -\infty$, $f(x) \to 0$.

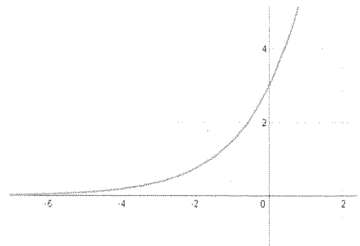
    EX: $f(x)=2^x$
  - **When 0<b<1 we have exponential decay.**

    End Behavior: As $x \to \infty$, $f(x) \to 0$. As $x \to -\infty$, $f(x) \to \infty$.

    EX: $f(x)=\left(\frac{1}{2}\right)^x$
  - **Coefficient's effect** $f(x)=a \cdot b^x$

    y-intercept is determined by the coefficient since when x=0,

    $$\begin{aligned} f(0) &= a \cdot b^0 \\ &= a \cdot 1 \\ &= a \end{aligned}$$

    EX: $f(x)=3 \cdot 2^x$

    This has a y-intercept of 3.
- **Inverse Functions –** functions that undo each other. The domain and range are reversed.
  - **Notation** $f^{-1}(x)$: We use an exponent of -1 to indicate the inverse function. This does NOT indicate a reciprocal or power of -1.
  - **Algebraically**, to find the inverse of a function, you exchange x and y and solve for the new y.

EX:

$$f(x)=y=\frac{2x-3}{5}$$

$$x=\frac{2y-3}{5}$$

$$5x=2y-3$$

$$5x+3=2y$$

$$\frac{5x+3}{2}=y=f^{-1}(x)$$

- **Graphically,** to find the inverse of a function, you reflect the original function across the line y = x.

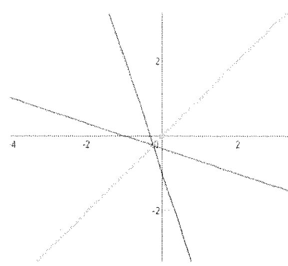
  EX:

  $$f(x)=y=-3x-1$$

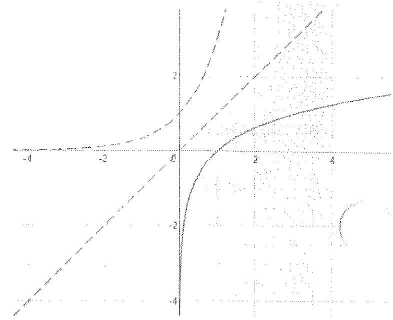
  $$x=-3y-1$$

  $$x+1=-3y$$

  $$f^{-1}(x)=y=\frac{x+1}{-3}$$



- **Logarithmic function** with base b is the inverse of the exponential functions with base b. A log is the exponent to which a base must be raised in order to obtain a given value.
    - **Graphically:** A log is the reflection of an exponential function over the line y=x.
    - **Algebraically:** A log is the inverse of an exponential function.

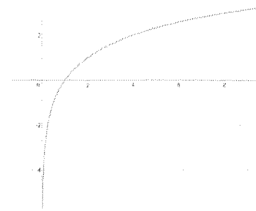


EX: $f\left(x\right)=\log_3 x$ is the same as $3^y = x$ which is a reflection over the line y = x of the function $3^x = y$.

- **When b>1:**

  End Behavior: As $x \to \infty$, $f\left(x\right) \to \infty$. As $x \to 0^+$, $f\left(x\right) \to -\infty$

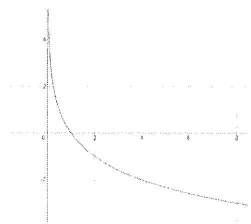
  EX: $y = \log_2 x$



- **When 0<b<1:**

  End Behavior: As $x \to \infty$, $f\left(x\right) \to -\infty$. As $x \to 0^+$, $f\left(x\right) \to \infty$

  EX: $y = \log_{\frac{1}{2}} x$



- **Common Log** has base 10:

  $$f\left(x\right)=\log x$$
  $$=\log_{10} x$$

- **Natural Log** has base e:

  $$f\left(x\right)=\ln x$$
  $$=\log_e x$$

- **Properties of Logs – The following properties hold for log and for ln.**
    - **Definition based:**

      $\log_b b^m = m \qquad \log_b 1 = 0 \qquad \log_b b = 1$

      $\ln e^m = m \qquad \ln 1 = 0 \qquad \ln e = 1$

    - **Product:**

      $\log_b mn = \log_b m + \log_b n$

      $\ln mn = \ln m + \ln n$

    - **Quotient:**

      $$\log_b \frac{m}{n} = \log_b m - \log_b n$$

      $$\ln \frac{m}{n} = \ln m - \ln n$$

    - **Power:**

      $\log_b m^n = n \log_b m$

      $\ln m^n = n \ln m$

    - **Change of Base:**

      $$\log_c a = \frac{\log_b a}{\log_b c}$$

    - **Inverses:**

      $b^{\log_b x} = x$

      $e^{\ln x} = x$

- **Compounding Interest** $FV = PV(1+r)^t$
  - **Simple interest (compounded annually):** $A(t) = P(1+r)^t$

    EX: You have \$500 to deposit into a savings account making 2% simple interest. How much money will be in your account in 4 years?

    $$A(t) = P(1+r)^t$$
    $$A(t) = 500(1+.02)^t$$
    $$A(4) = 500(1.02)^4$$
    $$= 541.21$$

    You will have earned \$41.21 in interest during the 4 years.

  - **Compounding Interest n times per year:** $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$

    EX: You have \$500 to deposit into a savings account making 2% interest that is compounded monthly. How much money will be in your account in 4 years?

    $$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$
    $$A(t) = 500\left(1+\frac{.02}{12}\right)^{12t}$$
    $$A(4) = 500\left(1+\frac{.02}{12}\right)^{12(4)}$$
    $$= 541.61$$

    You earned a total of \$41.61 in interest during the 4 years.

  - **Compounding Interest continually:** $A(t) = Pe^{rt}$

    EX: You have \$500 to deposit into a savings account making 2% interest that is compounded continuously. How much money will be in your account in 4 years?

    $$A(t) = Pe^{rt}$$
    $$A(t) = 500e^{.02t}$$
    $$A(4) = 500e^{.02(4)}$$
    $$= 541.64$$

    You earned a total of \$41.64 in interest during the 4 years.

- **Methods to solve exponential and logarithmic equations:**
  - For exponential equation...If possible, rewrite in form $b^x = b^y$ which can be simplified to $x = y$.

    EX:

    $$3^{x+4} = 27^2$$
    $$3^{x+4} = (3^3)^2$$
    $$3^{x+4} = 3^6$$
    $$x+4 = 6$$
    $$x = 2$$

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$x = y^2$, $\sqrt{x} = y$, $(\ )^2 = (\ )^2$, $x = y^2$

$x = 2^y$, $\log_2 : \log_2$, $\log_2 x = y$, $2^{\wedge} = 2^{\wedge}$, $x = 2^y$

Is it an exponent or log equation?

exponent → **Can you make the bases the same?**

yes →

1. Make the bases the same.
2. Set the exponents =

$$4^{x-2} = \left(\tfrac{1}{8}\right)^{2x-4}$$
$$(2^2)^{x-2} = (2^{-3})^{2x-4}$$
$$2^{2x-4} = 2^{-6x+12}$$

$2^3 = 2^x$

$$2x - 4 = -6x + 12$$
$$+6x + 4 = +6x + 4$$
$$\frac{8x}{8} = \frac{16}{8}$$
$$\boxed{x = 2}$$

no →

1. Log both sides of the equation (undo exponent) to get x out of the exponent.

$$3^x = 8$$
$$\log \quad = \log$$
$$\log 3^x = \log 8$$
$$x \frac{\log 3}{\log 3} = \frac{\log 8}{\log 3}$$
$$x = \frac{\log 8}{\log 3}$$
$$\boxed{x \approx 1.893}$$

log → **Do all the terms have a log?**

yes →

1. Condense both sides of the =
2. Exponentiate to cancel (undo) the logs and solve.

$$\log_5 x + \log_5 3 = \log_5 27$$
$$5^{\wedge}\ \log_5 3x = \log_5 27 = 5^{\wedge}$$
$$\frac{3x}{3} = \frac{27}{3}$$
$$\boxed{x = 9}$$

no →

1. Get the logs alone on one side of the =
2. Condense the logs
3. Exponentiate to change to exponential form.

$$\log_2(4x) - 3 = \log_2 5$$
$$-\log_2 5 + 3 = -\log_2 5 + 3$$
$$\log_2(4x) - \log_2 5 = 3$$
$$\log_2 \frac{4x}{5} = 3$$
$$2^{\wedge} \qquad = 2^{\wedge}$$
$$\frac{5}{4} \cdot \frac{4x}{5} = 2^3 \cdot \frac{5}{4}$$
$$\boxed{x = 10}$$

Check to see if all the logs are positive